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ABSTRACT

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Using Categorical Variables in Discriminant Analysis

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Abstract

Three methods of transforming unordered categorical response variables are described. One is a method using dummy binary variables. The second method analyzes all categorical variables simultaneously and is based on an eigenanalysis of frequency patterns scaled relative to within-groups variance, jointly developed by J. E. Overall and J. A. Woodward. The third method, independently developed by R. A. Fisher and H. O. Lancaster, analyzes each categorical variable separately and yields scale values so that the grouping variable and the categorical variable are maximally correlated. Results of analyzing two real data sets are used to make some tentative comparisons.

Using Categorical Variables in Discriminant Analysis

There are two aspects of discriminant analysis: description and prediction. That is, discriminant analysis techniques are useful in describing effects of grouping variables, and are also useful in predicting group membership of experimental units--both uses are usually made in the context of multiple response variables. The analysis techniques typically used by behavioral researchers are those available in computer program packages such as BMD, SAS, and SPSS. The discriminant analysis techniques used in most of these package programs implicitly call for input response measures that are based on at least an ordinal scale. In fact, some techniques theoretically call for multivariate normal distributions of response variables.

The robustness of normal-based discriminant analysis techniques to nonnormality has been studied to some extent--see Zhezhel (1968), Lachenbruch, Sneeringer, and Revo (1973), Krzanowski (1977), Koffler and Penfield (1979), Dillon (1979), and Beauchamp, Folkert, and Robson (1980). These studies were pretty much restricted to continuous variables and to two groups, and dealt basically with the prediction problem. Study, also, has been done with transformations of nonnormal continuous variables--see, e.g., Anderson and Richardson (1979), and Conover and Iman (1980).

Categorical Response Variables

The concern of the current paper is that of using categorical response variables in discriminant analysis. From a prediction standpoint, such variables have been dealt with directly, using a "density estimation" approach (see, e.g., Goldstein & Dillon, 1978, pp. 12-19; Wilcox, 1980;

Overall & Klett, 1972, Ch. 16). For situations involving many response variables with many categories, this approach is not without shortcomings, because some estimates may be based on little data.

Another way to handle categorical variables is to transform them to a form that is adaptable for normal-based analyses. [Terms that have been used in accord with such transformations are scaling, metricising, and calibration.] For two specific types of categorical variables there are "natural" transformations. In the special case of a two-category or Bernoulli variable, it has been shown (Bryan, 1961, p. 9) that optimal calibration involves a 0-1 assignment (see, also, Maxwell, 1961). Thus, for a variable such as Sex, the value for Male could be 0, and that for Female, 1. [See Krishnan (1973) for another approach with Bernoulli variables.] Another special case involves an ordered categorical variable. For this case there is evidence to conclude that "integer scaling" is near-optimal--see Kundert (1973, pp. 60, 89). For example, with the variable Socioeconomic Status, it is reasonable to assign 1 for "low," 2 for "middle," and 3 for "high." [See Johnson (1950) for another scaling approach with ordered categories.]

Of particular concern in this paper is the use of categorical response variables that have more than two unordered categories. [Henceforth in this paper, "categorical variable" is used in this sense.] Examples are marital status, college academic major, occupation, and disease diagnosis. Oft times data on such variables are obtained along with data on continuous variables. It may be desirable to transform the categorical variables so that the transformed scores along with the raw scores on the continuous variables can be used as input for a normal-based analysis.

The purposes of this study are: (i) to describe three procedures for transforming (unordered) categorical variables; and (ii) to illustrate the use of the three transformations on two sets of real data.

The design considered in this paper is the one-way layout (i.e., one grouping variable) with p response variables, k groups, and a total of $N = \sum_{g=1}^{k} n_g$ experimental units.

Transformations of Unordered Categorical Variables

Dummy Variable Method

One way of dealing with a categorical variable is to transform it to Bernoulli form by means of defining dummy binary variables (Lachenbruch, 1975, pp. 54-55). For example, $X_1 = 1$ if single, 0 otherwise; $X_2 = 1$ if married, 0 otherwise; $X_3 = 1$ if divorced or separated, 0 otherwise, may be used in place of Y = marital status (0 = single, 1 = married, 2 = divorced or separated, 3 = widowed). For this example, the transformation produces three variables from the one four-category variable. If there are q categorical variables, the jth one having c_j categories, the dummy variable transformation would yield $\sum_{j=1}^{q} = (c_j - 1)$ Bernoulli variables.

Overall-Woodward Method

A second transformation is due to Overall and Woodward (1977). With this procedure, all q categorical variables are analyzed simultaneously. The method of analysis is developed by analogy to that of deriving linear discriminant functions (LDFs). It is essentially a principal component analysis of frequency patterns across all of the categorical variables. Suppose there are q categorical variables, c_j categories for variable Y_j, yielding a total of $C = \sum_{j=1}^{q} c_j$ categories. The C x N incidence matrix, containing the category frequencies, is used to arrive at a C x k matrix, Z, of proportions of units in each of the k groups that fall in each of

the C categories, deviated about the unweighted mean of each category proportion across all k groups. That is, a column of Z is defined by the C x 1 vector

$$\underline{p}_g - \bar{\underline{p}} ,$$

where $\underline{p}_g$ is the vector containing the proportion of units in the gth group that fall in each category, and $\bar{\underline{p}}$ is the vector containing the unweighted mean of the category proportion vectors across all k groups. The Z matrix then transformed to $\tilde{Z} = D^{-1}Z$, where D is the C x C diagonal matrix containing the positive square roots of the average (over the k groups) within-groups variances for the 0-1 observations in the C categories.

The r principal components of the C x C matrix, $\tilde{Z}\tilde{Z}'$, are then found. Let $\tilde{A}$ be the C x r matrix of the r components (i.e., eigenvectors) of $\tilde{Z}\tilde{Z}'$. The elements of $A = D^{-1}\tilde{A}$ are the raw-score category weights that define the r components. Thus, each unit has r values for analysis input. These r values are simply sums of the elements of A corresponding to categories in which the unit belongs. For this transformation, r new response "variables" are produced to represent the q categorical variables.

Fisher-Lancaster Method

The third and final transformation to be considered is one that yields scale values for each category separately for each variable. For q categorical variables, there will result q sets of scale values. The procedure was independently developed, from different points of view, by Fisher (1940) and Lancaster (1957). The scale values are found, in a Fisher sense, so that the group discriminatory power of a given variable is maximized; in a Lancaster sense, the leading canonical correlation between the grouping variable and the categorical variable is maximized.

[A detailed presentation of the two views is given by Kundert (1973, pp. 11-31), while Bryan (1961) and Henschke, Kossack, and Lissitz (1974) present a calculation routine.] For a categorical variable Y_j, a c_j x k table of frequencies is constructed. Then a c_j x c_j matrix, M, is formed, entries of which are functions of entries of the c_jx k incidence matrix:

$$m_{ii'} = \left[\sum_{g=1}^{k} \frac{n_{ig} n_{i'g}}{n_{.g}} - \frac{n_{i.} n_{i'.}}{N} \right] (n_{i.} n_{i'.})^{-\frac{1}{2}} ,$$

where n_{ig} is the frequency in category i for group g. The largest eigenvalue of M and the associated eigenvector, $\underline{v}$, is obtained. Scale values for category i of Y_j are obtained using

$$w_{ji} = v_{ji} / \sqrt{n_{i.}} .$$

[Henschke et al. (1974, p. 110) left off the radical sign.] The w_{ji} values are the optimum category values that are used in "scoring" the given categorical variable. That is, a unit in the ith category of variable Y_j is given a "score" of w_{ji}. The process is repeated for all q variables, getting q sets of w values.

Computer Programs

The two latter transformations of categorical variables were accomplished via computer programs written by the second author. These programs, which utilize SAS's PROC MATRIX, are available upon request.

Comparison Assessment

A comparison of results of the three transformations may be made on the basis of group separation and on the basis of group membership prediction accuracy. Both bases will be utilized in this paper, to the

extent where they are reasonably appropriate, when transformed categorical variables are used along with untransformed continuous variables in a normal-based analysis. Group separation is assessed using the Wilks lambda statistic, if it is tenable that the k population covariance matrices can be assumed to be identical. Prediction accuracy is assessed using percents of correct group classifications, "hit rates." The classification rule used is linear normal with priors based on group sizes (see, e.g., Overall & Klett, 1972, p. 348). For comparison purposes, external hit rates are used. The external analysis used is the leave-one-out (L-O-O) method (Lachenbruch, 1975, p. 32) via BMD's P7M program. A linear external classification analysis is used regardless of the tenability of the equal covariance matrix condition--there is some evidence (Huberty & Curry, 1978) that linear rules yield more reliable results than do quadratic rules.

Data Sets

Real data are utilized to illustrate the three procedures and to make some preliminary comparisons. Data Set 1 is taken from Afifi and Azen (1979, pp. 23-27). The grouping variable used is Clinical Status with three levels: hypertensive/rheumatic and possible/potential heart disease ($n_1 = 13$), hypertension without heart disease ($n_2 = 15$), and normal ($n_3 = 60$) -- $N = 88$. Two response measures have underlying continua: diastolic blood pressure and percent of serum cholesterol. Two measures are on unordered categorical variables: examining physician (MD, with five categories) and ischemic heart disease diagnosis (DD, with four categories). It is these two categorical variables that are of primary focus in this paper; they are transformed so as to be more "compatible" with the two continuous variables.

Data Set 2 is a subset of a large set of data collected via a nation-wide survey of 1980 high school seniors--the High School and Beyond Study sponsored by the National Center for Education Statistics. The grouping variable used is Participation in Church Activities, with three levels: no participation ($n_1 = 99$), participation ($n_2 = 60$), and participation as a leader ($n_3 = 40$). Two response measures have underlying continua: locus of control composite scale score, and self-concept composite scale scores. Two unordered categorical variables used are: occupational aspiration at age 30 (OA, with 5 categories), and main activity in year after high school (PA, with 4 categories).

Results

Data Set 1

Univariate analyses were conducted on the four response variables, using the analysis of variance F statistic for diastolic blood pressure (DBP) and percent of serum cholesterol (SC), and the likelihood ratio chi-squared statistic for examining physician (MD) and ischemic heart disease diagnosis (DD). These analyses tested for equality of three mean or three proportion vectors. The results are: for DBP, $F(2,85) = 10.36$, $p < .001$; for SC, $F(2,85) = 4.68$, $p < .025$; for MD, $\chi^2(8) = 19.48$, $p \doteq .01$; and for DD, $\chi^2(6) = 7.94$, $p \doteq .24$.

The weights applicable to variable categories obtained via the Overall-Woodward procedure, where the categorical variables are analyzed simultaneously, are presented in Table 1. Also found in this table are

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Insert Table 1 about here

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the category weights obtained via the Fisher-Lancaster procedure, where each categorical variable is analyzed separately.

The separation and classification results are presented in Table 2.

Insert Table 2 about here

Although the Wilks lambda value is given for the dummy variable approach, its appropriateness may be questioned since the equal covariance matrix condition was judged untenable. [In fact, covariance matrix singularity resulted.] This condition was, however, judged tenable with both the Overall-Woodward and Fisher-Lancaster transformations. The O-W transformation of the two categorical variables along with the two continuous variables yielded a little better separation than did the F-I transformation. The leave-one-out classification analysis results consistently favored the O-W approach, for two separate-group and the total-group hit rates. Parenthetically, it may be noted the classification rules used with the three transformations yielded total-group hit rates not appreciably better than $60/88 \doteq .68$, that yielded by the rule, "assign all units to Group 3." A total-group hit rate of 0.70 is, however, appreciably greater than the chance hit rate of 0.52 based on all three groups with the prior probability of membership in group g estimated by n_g/N.

Data Set 2

Univariate analyses on the four response variables yielded the following results: for locus of control (LC), $F(2,196) = 0.73$, $p \doteq .50$; for self-concept (SC), $F(2,196) = 3.56$, $p < .05$; for occupational aspiration (OA), $\chi^2(8) = 8.08$, $p \doteq .43$; and for post-high school activity

(PA), χ^2 (6) = 14.86, $p \doteq .02$.

The weights applicable to variable categories via the Overall-Woodward procedure are presented in Table 3. Also found in this table are the

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Insert Table 3 about here

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category weights obtained via the Fisher-Lancaster procedure.

The separation and classification results are presented in Table 4.

Insert Table 4 about here

With this data set, the equal covariance matrix condition is tenable for all three transformations. Separation of groups with the dummy variable approach is somewhat less than that with the O-W and F-L approaches. Hit rates for the two larger groups are a little lower with the dummy variable approach than with the O-W and F-L approaches. In terms of group separation and classification accuracy, there is not much to choose between the O-W and F-L approaches. As with Data Set 1, the three classification rules do no better than the rule that assigns all units to the largest group, but do somewhat better than the three-group chance hit rate of 0.38.

Discussion

Procedures for carrying out a predictive discriminant analysis when all of the response variables are categorical have been advanced (e.g., Andrews & Messenger, 1973; Glick, 1973; Hills, 1967) and, to some extent, studied (e.g., Dillon & Westin, 1982; Gitlow, 1979; Wilcox, 1980). These procedures are not entirely satisfactory for two reasons: (1) There are times when researchers want to study group differences, and, thus, apply

descriptive discriminant analysis techniques either instead of or in addition to predictive discriminant analysis techniques; and (2) It is often the case that only some of the response variables are categorical (with, for current concerns, unordered categories), while others are continuous and measured on at least an interval scale. [Krzanowski (1980) presents a procedure for analyzing a mixture of continuous and categorical variables, but basically for the purpose of group membership prediction.] A means of handling unordered categorical response variables so that group differences with respect to these variables can be studied, and so that the categorical variables can be analyzed in conjunction with continuous variables, is to transform the former in a manner so that all response variables can be analyzed simultaneously (using the widely available computer program packages).

This paper presents two such transformation procedures, and illustrates the use of each with real data sets. One is termed the Overall-Woodward transformation, and the second the Fisher-Lancaster transformation. [It may be noted that the F-L transformation has the same goal as that of the computer program algorithm discussed by Young, deLeeuw, and Takane (1976), namely, to maximize a canonical correlation.] A third procedure, that of utilizing a dummy binary variable transformation of the categorical variables, is also illustrated.

Conclusions about comparisons of the three procedures using the two real data sets can at best be considered tentative. One tentative conclusion that might be reached is that a scaling of unordered categorical variables other than that based on the use of dummy binary variables ought to be utilized. [That the use of dummy variables is not recommended is

in opposition to a recommendation by Dillon and Westin (1976).] In addition to the comparative results obtained herein, this conclusion is supported by the fact that the number of dummy variables needed can become unwieldy, and conditions needed to assess group separation may very well not be met. The advantage of simplicity that might be given to the dummy variable approach is essentially removed with the availability of computer programs for other transformations--those written by the second author of this paper, as well as those by Young et al. (1976) and by Overall (1974).

More study, obviously, needs to be done in comparing these three approaches, particularly the Overall-Woodward and Fisher-Lancaster procedures. More real data sets need to be studied, with varying numbers of groups, and with categorical variables yielding different degrees of group separation and classification accuracy. Or, perhaps, these procedures could be studied using simulated data sets.

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Table 1

Category Weights for
Data Set 1

Overall-Woodward

Variable	Category	Weights Vector 1	Vector 2
DD	1	-.796	.951
	2	.066	-1.426
	3	.376	.982
	4	-.828	.238
MD	1	-.679	-1.276
	2	-.183	-1.363
	3	.613	.153
	4	-.864	-.889
	5	1.227	-1.058
Eigenvalues:		1.540	0.469

Fisher-Lancaster

Variable	Category	Weight
DD	1	.552
	2	-2.164
	3	.221
	4	-.148
MD	1	-.467
	2	1.036
	3	-1.601
	4	.902
	5	1.782

Table 2

Separation and Classification Results for
Data Set 1

Dummy Variable

Separation: $\Lambda \doteq .530$

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	4 [.31]	1	8	13
	2	1	7 [.47]	7	15
	3	2	7	51 [.85]	60
		7	15	66	88

Total-group L-0-0 hit rate is 0.70.

Overall-Woodward

Separation: $\Lambda \doteq .556$ [$F(8,164) = 6.99$, $p < .001$]

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	4 [.31]	1	8	13
	2	0	9 [.60]	6	15
	3	0	6	54 [.90]	60
		4	16	68	88

Total-group L-0-0 hit rate is 0.76.

Fisher-Lancaster

Separation: $\Lambda \doteq .605$ [$F(8,164) = 5.86$, $p < .001$]

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	4 [.31]	1	8	13
	2	0	5 [.33]	10	15
	3	1	6	53 [.88]	60
		5	12	71	88

Total-group L-0-0 hit rate is 0.70.

NOTE: Separate-group L-0-0 hit rates are given in brackets.

Table 3

Category Weights for
Data Set 2

Overall-Woodward

Variable	Category	Weights Vector 1	Vector 2
OA	1	.402	.466
	2	1.175	-1.204
	3	.035	-.115
	4	-.295	-.031
	5	.285	2.886
PA	1	-1.285	.478
	2	1.816	.603
	3	.037	-.310
	4	.290	-.244
	Eigenvalues:	.456	.035

Fisher-Lancaster

Variable	Category	Weight
OA	1	.623
	2	.783
	3	-1.677
	4	-.281
	5	1.072
PA	1	.948
	2	-1.027
	3	-.245
	4	1.607

Table 4

Separation and Classification Results for Data Set 2

Dummy Variable

Separation: $\Lambda \doteq .866$ [$F(18,376) = 1.55$, $p \doteq .10$]

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	74 [.75]	19	6	99
	2	43	10 [.17]	7	60
	3	25	9	6 [.15]	40
		142	38	19	199

Total-group L-0-0 hit rate is .45

Overall-Woodward

Separation: $\Lambda \doteq .876$ [$F(8,386) = 3.32$, $p < .001$]

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	78 [.79]	17	4	99
	2	38	16 [.27]	6	60
	3	23	11	6 [.15]	40
		139	44	16	199

Total-group L-0-0 hit rate is .50.

Fisher-Lancaster

Separation: $\Lambda \doteq .876$ [$F(8,386) = 3.29$, $p < .001$]

Classification (L-0-0):

		Predicted Group 1	2	3	
Actual Group	1	76 [.77]	18	5	99
	2	38	16 [.27]	6	60
	3	23	11	6 [.15]	40
		137	45	17	199

Total-group L-0-0 hit rate is .49.

Note: Separate-group L-0-0 hit rates are given in brackets.